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# ON LEONHARD EULER'S THEORY OF MUSIC

## Leonhard Euler zeneelméleti munkája

(Emanuel Handmann festménye, 1753)

## Absztrakt

A cikk a híres svájci tudós, Leonhard Euler** műveiben kifejezett zeneelméleti nézeteinek elemzésével foglalkozik. (Miután a tudós már hosszú ideje Oroszországban dolgozott.)

E nézetek, történelmileg megmagyarázható korlátaik ellenére is számos figyelemreméltó heurisztikus vonással rendelkeznek, amelyek egészen napjainkig aktuálissá teszik. Az írás rámutat Euler visszatérésére az információ zenében megjelenő fogalmához, elemzi az ő sajátos harmónia probléma interpretációját, valamint a hangmagasság és a zene ritmikus szervezése közötti kapcsolat perspektivikus jelentőségét, amelyet a tudós periódikus folyamatok széles körében vizsgált.

**Abstract:** The article is devoted to the analysis of the theoretical views on music expressed in the works by the famous Swiss scientist (having been working in Russia for a long time) Leonhard Euler (1707 – 1783). Despite the historically explainable bounds of these views, they proved to be remarkable for numerous features of high heuristic significance that are important up to nowadays. The text of the article points out Euler's rapprochements to the notion of information in music, his original interpretation of harmony problems, as well as of the relations between pitch and rhythm organization placed in the more general series of periodical processes.

**Keywords:** Leonhard Euler, music theory, harmony, rhythm, theory of information, music computer technologies.

The attitudes given by musicologists to the theoretical works by Leonhard Euler (1707 – 1783) devoted to music and some related fields of research [1 – 8] may be qualified as very contradictory. Some authors tend to see in Euler's theoretical discourses on music an example of 'misfortune having overtaken the great scientist' [9 – 10]. The grounds for such opinions probably lie in the fact that a considerable set of propositions presented in Euler's works on music theory should not be interpreted only in their narrowest sense, as a 'definite solution' of musicological problems that had stimulated the appearance of these propositions in Euler's times. 'Hypercritical' interpretations of Euler's musicological heritage often ignore the complicated character of research processes and, as a result, represent the real picture of development proper to musicology in a scanty, impoverished form. In this respect, much more realistic is the position of such authors that, being far from denying the features of limitedness in Euler's music theory, regard it, on the whole, as a positive phenomenon the main validity of which is determined by raising a number of fundamental musicological problems, as well as by the tendency to the consequently scientific methods of analyzing the phenomena being studied (the tendency that is also proper to other Euler's works) [11 – 13].

The problem importance of Euler's theory is evidently displayed in his attempt of systematizing the interrelations of sounds and their combinations on the basis of the category of 'pleasantness' (or 'euphony', *suavitas*) [1]. Despite the historically explainable incoherency of Euler's abstract mathematical approach to the essence of this category (belonging to the sphere of social psychology in its nature [14]) and its individual manifestations in music, the attempt made by him is undoubtedly remarkable as bearing in it the more general idea of integrative index of complexity (respectively – of regularity in order) concerning the logical organization in music and given in the respect of perceiving this organization by an individual subject. This idea stands close to the later notion of information that is essential for aesthetic evaluation of various phenomena [15] and the significance of that (as well as of the Eulerian category of 'pleasantness') is not limited to purely logical sphere, it is much broader in its meaning and richer in its content.

Besides that notion – philosophical in its general essence – the aesthetical aspects of which concerning music had been proved very soon after its suggesting [16 – 18], the theoretical thought on music proposed (mainly – in the 20th century) a number of similar integrative characteristics, though more 'local' in their meaning, such as 'density' [19 –20], 'tension' [21 – 23], 'sound temperature' [24] of musical structures and so on. Some of these propositions (e. g. 'tension' of pitch combinations and their systems) played an important role in comprehending the new logical regularities in music of the 20th century, assisting thereby to the process of 'contributing clearness and proportionality' (according to well-known Sergey Taneiev's words [25]) into their practical application. It is evident that the structural principles of such generalizations (being not equal in their individual contents) demonstrate the features of similarity with Euler's analogous attempt, and this gives the grounds for evaluating the latter as containing a perspective of outstanding historical scale and importance. The main direction of this perspective does not totally coincide with some of initial premises of Euler's theory, guided by the notion of simplicity (and not of interrelations between simplicity and complexity) as expression of ‘euphony’ and ‘perfection’ of pitch combinations [1, ch. 2, § 8, 13]; but the tendency to co-ordinate the theoretical calculations with the data of practical experience leads Euler logically to the conclusion of optimal euphony as occupying a certain middle place between the minimum and the maximum of complexity of that kind of phenomena accessible for human perception[1, ch. 8, § 13 – 16].

Similar worth-while possibilities may be found in some other Euler’s ideas concerning the logical organization of music (resp. the functions of the ‘pleasantness’ category). In particular, to this group belongs the idea of horizontal and vertical identity in pitch [1, ch. 5, § 3] that supposes a relative independence of the indices of ‘pleasantness’ from the factor of time. The meaning of this idea proposed by Euler has two different aspects that embrace both the field of logics and that of psychology of musical perception. First of all, this proposition corresponds to really observed psychological phenomenon of simultaneous integrating (simultanizing) the musical structures ([26, p. 245]; it should be noted that in this case the final result, according to Euler, is not a sound but an abstract numerical index of ‘pleasantness’). Secondly, this point of Euler’s theory constitutes a kind of parallel with some historical processes having taken place (though mainly in ‘peripherical’ form far from the focus of observation) in music of the 17$^{th}$ and 18$^{th}$ centuries before being advanced to the forefront in the 20$^{th}$ century. The main results of these processes were polychord settings in harmony [27 – 28] and developed forms of ‘free’ counterpoint that assume – in correspondence with the peculiarities of individual artistic concepts – any necessary number of voices and any necessary simultaneous combination (resp. interrelation in pitch) of them [29]. In this connection, Euler’s using the thorough-bass figures, having no strict differences between so-called ‘chordal’ (self-dependent) and ‘non-chordal’ (bound up) voice combinations, and, thus, supposing logically the possibility (translated into reality in later history of music [30]) of transforming the latter into self-dependent – is very symptomatic•). It is also remarkable that some of such combinations with a relatively high index of complexity (‘decreasing euphony’, according to the initial premises of Euler’s theory) analyzed by Euler are very similar to acciaccaturas used in musical practice of his times (e. g. in keyboard sonatas by Domenico Scarlatti). At the same time, it is quite clear that Euler’s index of simultaneous ‘pleasantness’ must be supplemented by the corresponding index characterizing the peculiarities of transition between the structure elements and, thus, belonging to the sphere of ‘musical time’ that is different from the ‘spatial’ aspect of music manifestations.

The necessity of such index becomes realized by Euler himself when studying the processes of modulation and formation of general composition structures that presuppose the interaction of complex pitch systems [1, ch. 13 – 14]. It is noteworthy that, while characterizing logical and psychological details of transition (modulation) from one key to another, Euler uses a matrix form [1, p. 262] known from later history of music theory in connection with studying different levels of logical organization in music – from transitions between individual sounds [31] to interactions of vast multi-dimensional sound complexes [24, ch. 3] – as well as in connection with studying musical synaesthesias [32]. One of the most remarkable features of Euler’s modulation matrix is an element of game theory (evaluation of modulations as ‘easy’, ‘difficult’ or ‘unattainable’) that is interesting not only as a historical fact of musicology but also as that of mathematics (in which the period of active formation of game theory falls on the 20th century).

Euler’s propositions concerning musical pitch system are based on non-tempered tuning, and this feature of his theoretical views on music that could seem to be archaic in times of the past reveals its worth-while possibilities in the light of

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•) This characteristic feature of Euler’s theory is in conformity with another idea proposed by Euler [2] – that of correspondence between the consonance-dissonance opposition and the degrees of complexity of pitch combinations (measured by frequency ratios). In its essence, this latter proposition forms the basis for idea of relativity of consonance and dissonance that became important for the music of the 20th century (see e. g. [28], [30]).

microtonal aspects of contemporary music (attainable, in particular, with the assistance of electronic and computer music devices). Moreover, Euler proposed also the system of measuring intervals in binary logarithms of frequencies – in other words, in parts of octave [1, ch. 7, § 4] – that in essence is of the same kind with the principles of equal temperament. Thereby, Euler in fact was taking into consideration the both principles of forming the pitch systems that played an important role in history of music (being reflected correspondingly in history of music theory) – namely, those of addition and of division in relations of the system elements. The specific form of relation between these principles themselves in Euler's views characterizes the researcher's individual position on this problem. In this connection, a kind of the 24-degree temperament using the 'natural minor seventh' (frequency ratio 7:4) as one of the basic intervals [6] should also be noted among Euler's contributions to this field of music theory.

In contrast to Euler's attempt of studying pitch organization in music, his works on musical rhythm, as well as on the problem of correlation between light and sound [3 – 8], never were a subject for an actual discussion. Euler's theoretical remarks on rhythm problems expressed by the researcher in some of his early (unpublished) works, remained unknown to the broader sections of readers (an idea of Euler's views on this subject may be formed on the basis of information given in the article [13]). And, as to Euler's analogies between light and sound (partly – also smell), the authors of musicological studies note his (also characteristic historically) purely physical approach to the problem ('an ear, eye, even a nose, as certain highly perfect devices of research in a physicist's laboratory' [11, p. 24]). Probably, these features of Euler's works formed the reason for having them been left beyond the 18th-century famous discussion on the nature of sight-and-hearing synaesthesias (in connection with the problem of colour-and-music synthesis) inspired by the researcher's contemporary Louis-Bertrand Castel, though Euler himself showed some interest to Castel's ideas [8]. Nevertheless, the idea of similarity in structure of pitch and rhythmical systems deduced by Euler from belonging of the both phenomena to the category of periodical processes (expressed in a general way in his main treatise on music theory [1]) stimulated later (though not directly) a considerable number of studies and theoretical systematizations of musical rhythm that have resulted in essential enlarging of this field of musical expression (e. g. the theoretical concepts by Henry Cowell [28], Karlheinz Stockhausen [33], Ludwik Bielawski [34] a. o.). Bielawski's theory, for example, constitutes an attempt of constructing the universal (and embracing also analogies to sound adhered by Euler) scale of periodical processes going far beyond the musical field in the proper sense, but, despite the evident marks of simplification in its content (as well as in the analogous scale of systematizing spatial measuring units) it is remarkable as a kind of 'musicological breakthrough' to the spheres of other research disciplines, giving worth-while ideas in the other fields of artistic (and human in general) activities. Initial premises for that breakthrough, however, are available in Euler's theoretical works on music, as expressed by the researcher himself: *Praeterea quoque in omnibus aliis rebus, in quibus decorum et ordo inesse debet, haec tractatio magnam habebit utilitatem, si quidem ea quae ordinem constituunt, ad quantitates reduci numerisque exprimi possunt; sicut in Architectura, in qua decori gratia requiritur, ut omnes aedificii partes ordine, qui percipi possit, sint dispositae* [1, p. 43][•)].

In conclusion, the peculiarities of musical theory suggested by Euler give the grounds for speaking about its high heuristic significance essentially corroborated by posterior development both in theory and practice of music. In the general picture of historical development of theoretical thought on music Euler's innovative ideas occupy an important place. And it may be said with good reason that more active approach to these ideas would bring new fruitful theoretical and practical results also in the sphere of musical education and upbringing.

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•) Besides that, also in all other spheres, where order and beauty is needed, this approach may be very useful – certainly, if the matter put in order may be reduced to quantity and expressed in number; for example, in architecture, where the beauty requires for all the parts of the building to be disposed in order accessible for recognizing (Lat.).

A number of pedagogical experiments concerning the subject of this article (lectures, seminars, practical academic tasks) have been implemented in the process of holding the courses Mathematics and Computer Science at the Department of Music of the Herzen State Pedagogical University of Russia and the optional course Mathematical Methods of Research in Musicology for students of the Rimsky-Korsakov State Conservatory of St. Petersburg. The ideas of Leonhard Eulers' book having been analyzed in the article constituted the basis of practical academic exercises in musical mathematics within the limits of the courses. Among such exercises, there is interpretation of musical settings in form of set algebra operations, studies in practical modeling of regularities proper to the statistical distribution of characteristics of musical logic, calculation of information meaning of various manifestations of these regularities with application of well-known formulae of theory of information, etc. Moreover, some complex cases of this kind having been analyzed by Euler may appear in the role of optional elements of the course subjects, being introduced eventually, e. g. in connection with the specific problems that might arise in the process of students' research and practical creative work concerning the basic academic disciplines of music. These problems are analyzed more extensively by the authors of the article in the following works [35 – 37 a. o.].

The ideas stated in the proposed article found their more complete implementation in the academic course entitled 'Information Technologies in Music' that is one of the basic courses for students of the Herzen University Music Department specializing in the direction 'Artistic Education' within the limits of professional education profiles 'Music Computer Technologies' and 'The Art of Music'. For implementing the program of this course, the authors of the article have prepared the educational text-book *Information Technologies in Music* [38 – 43 a. o.] that contains the analyses of the interaction principles and forms concerning music, mathematics and computer science in their historical development (including its contemporary stage), as well as some recommendations for academic courses devoted to application of mathematics and information technologies in the fields of musicology and practical composition.

## References:

1. Euler L. Tentamen novae theoriae musicae ex certissimis harmoniae principiis delucide expositae. Petropoli: Academiae Scientiarum, 1739.

2. Euler L. Dissertatio physica de sono. Basileae: Thurnisiis, 1727.

3. Euler L. Nova theoria lucis ac colorum // Leonhardi Euleri opuscula. Varii argumenti. Berolini: A. Haude et J. C. Speneri, 1746. P. 169 – 244;

4. Euler L. Conjectura physica circa propagationem soni ac luminis. Berolini : A. Haude et J. C. Speneri, 1750.

5. Euler L. Conjecture sur la raison de quelques dissonances généralement reçues dans la musique // Mémoires de l'Académie des Sciences de Berlin, 20 (1764). Berlin, 1766. P. 165 – 173 ;

6. Euler L. Du véritable caractère de la musique moderne // Ibid. P. 174 – 179 ;

7. Euler L. De harmoniae veris principiis per speculum musicum representatis // Novi commentarii Academiae Scientiarum Petropolitanae, 18 (1773). Petropoli, 1774. P. 330 – 353.

8. Euler L. Lettres à une Princesse d'Allemagne sur divers sujets de physique et de philosophie. V. 1 – 3. St.-Pétersbourg: Académie Impériale des Sciences, 1768 – 1772.

9. Chevaillier L. Les théories harmoniques // Encyclopédie de la Musique et Dictionnaire du Conservatoire. 2$^{e}$ Partie. Paris : Librarie Delagrave, 1925. P. 519 – 579.

10.Hertzman E. V. Leonhard Euler i istoria odnoi muzykal'no-matematicheskoi idei (Leonhard Euler and history of a certain mathematical idea in music) // Razvitiye idey leonarda Eulera i sovremennaya nauka (The development of Leonhard Euler's ideas and contemporary science). Moscow : Nauka, 1988. P. 321 – 332.

11. Ossovsky A. V. Muzykal'no-esteticheskiye vozzreniya, nauka o muzyke i muzykal'naya kritika v Rossii v XVIII veke (Outlooks in aesthetics of music, musicology and music criticism in Russia in the 18$^{th}$ century) // Ossovsky A. V. Izbrannyie stat'i, vospominaniya (Selected articles, memoirs). Leningrad : Sovietsky kompozitor, 1961. P. 19 – 106.

12. Cerliuk-Askadskaya S. S. Muzykal'no-teoreticheskoe nasledie Leonharda Eulera i stanovlenie iego kontceptsii teorii muzyki (Leonhard Euler's heritage in music theory and

formation of his theoretical conception in the sphere of music). Summary of the Ph D dissertation (Art Criticism, the Art of Music). Kiev, 1987.

13. Cerliuk-Askadskaya S. S. Muzykal'no-teoreticheskie rukopisi Leonharda Eulera i stanovlenie iego kontceptsii teorii muzyki (Leonhard Euler's manuscripts on music theory and formation of his theoretical conception in the sphere of music) // Razvitiye idey leonarda Eulera i sovremennaya nauka (The development of Leonhard Euler's ideas and contemporary science). Moscow : Nauka, 1988. P. 333 – 344.

14. Zaripov R. H. Tvorchestvo v algoritmicheskikh ramkakh (Creative activities within the limits of algorithms) // Siuita dla personal'nogo kompiutera s orkestrom (Suite for personal computer and orchestra). Moscow: Znanie, 1989. P. 3 – 25.

15. Kryukovsky N. I. Kibernetika i zakony krasoty (Cybernetics and the principles of beauty). Minsk: Byelorussian State University, 1977.

16. Moles A. La théorie de l'information et la perception esthétique. Paris : Flammarion, 1958.

17. Pierce J. R. Symbols, Signals and Noise. The Nature and Process of Communication. New York: Harper & Brothers, 1961.

18. Zaripov R. H. Kibernetika i muzyka (Cybernetics and music). Moscow: Nauka, 1971.

19. Schillinger J. The Schillinger System of Musical Composition. V. 1 – 2. New York: Carl Fischer, 1946.

20. Křenek E. Extents and Limits of Serial Techniques // The Musical Quarterly, 1960, No. 2. P. 210 – 232.

21. Kurth E. Die romantische Harmonik und ihre Krise in Wagners Tristan (1920). Berlin: Max Hesse, 1923.

22. Hindemith P. Unterweisung im Tonsatz. T. 1. Mainz: B. Schott's Söhne, 1940.

23. Ogolevetz A. S. Vvedeniye v sovremennoye muzykal'noye myshleniye (Introduction into contemporary musical thought). Moscow & Leningrad: Muzgiz, 1946.

24. Xenakis I. Musiques formelles // La Revue musicale, No. 253/254. Paris, 1963. 232 p.

25. Taneiev S. I. Materialy i dokumenty (Materials and documents). V. 1. Moscow: AN SSSR (The USSR Academy of Sciences), 1952. P. 233.

26. Teplov B. M. Psikhologia muzykal’nykh sposobnostei (Psychology of musical abilities). Moscow & Leningrad: APN RSFSR (Academy of Pedagogical Sciences of the Russian Federation), 1947.

27. Milhaud D. Polytonalité et atonalité // La Revue musicale, 1923, No. 4. P. 29 – 44.

28. Cowell H. New Musical Resources. New York: Alfred A. Knopf, 1930.

29. Teoria sovremennoi kompozitsii. (Theory of contemporary composition). Moscow: Muzyka, 2005.

30. Schoenberg A. Harmonielehre. Leipzig: Peters, 1977.

31. Roiterstein (Reuterstein) M. I.Graf i matritsa kak instrumenty ladovogo analiza (Graph and matrix as instruments for analysis of modes) // Muzykal’noe iskusstvo i nauka (Musical art and science). 2nd issue. Moscow: Muzyka, 1971. P. 175 – 189.

32. Galeyev B. M. Problema sinestezii v iskusstve (The problem of synaesthesia in art) // Iskusstvo svetiaschikhsia zvukov (The art of luminous sounds). Kazan: Kazan Aviation Institute, 1973. P. 67 – 88.

33. Stockhausen K. …wie die Zeit vergeht // Stockhausen K. Texte. Bd. 1.Koln: DuMont Schauberg, 1962. S. 99 – 139.

34. Bielawski L. Strefowa teoria czasu i jej znaczenie dla antropologii muzycznej (The strata theory of time and its meaning for anthropology of music). Kraków: PWM, 1976.

35. Gorbunova I.B., Zalivadny M.S. Leonhard Euler’s theoretical views on music: actual meaning and some perspectives [Muzykal'no-teoreticheskie vozzrenija Leonarda Jejlera: aktual'noe znachenie i perspektivy]. Pushkin Leningrad State University Journal [Vestnik Leningradskogo gosudarstvennogo universiteta im. A.S. Pushkina. Nauchnyi zhurnal], 2012, no 4, vol. 2, pp. 164-172.

36. Gorbunova I.B., Zalivadnyj M.S. On mathematical methods in music theory and musical education [O matematicheskih metodah v issledovanii muzyki i podgotovke muzykantov]. Music Scholarship. Russian Journal of Academic Research in Music [Problemy muzycalnoy nauki]. 2013, no. 1(12), pp. 272-276.

37. Gorbunova I.B., Zalivadnyj M.S., Tovpich I.O. [Kompleksnaja model' semanticheskogo prostranstva muzyki i perspektivy vzaimodejstvija muzykal'noj nauki i sovremennogo muzykal'nogo obrazovanija] Integrative model for semantic space of music and prospects of interaction between the science of music and modern music education // Nauchnoe mnenie / Sankt-Peterburgskij universitetskij konsorcium, 2014,

no. 8, pp. 238-249. Nauchnoe mnenie [The Scholarly Opinion] Nauchnyi Zhurnal [A Research Magazine]/ Sankt-Peterburgskskiy universitetskiy konsortsium [The St. Petersburg University Consortium]

38. Gorbunova I.B. Information technologies in music. Vol. 1. Architectonics of musical sound. Textbook [Informatsionnye tekhnologii v muzyke. T. 1: Arkhitektonika muzykal'nogo zvuka: Uchebnoe posobie]. St. Petersburg, The Herzen State Pedagogical University of Russia Publ., 2009. 175 p.

39. Gorbunova I.B. Information technologies in music. Vol. 2. Music synthesizers. Textbook [Informatsionnye tekhnologii v muzyke. T. 2: Muzykal'nye sintezatory: Uchebnoe posobie]. St. Petersburg, The Herzen State Pedagogical University of Russia Publ., 2010. 205 p.

40. Gorbunova I.B., Pankova A.A. [Komp'juternaja muzyka. T. 1: Komp'juternoe muzykal'noe tvorchestvo: uchebnoe posobie] – St. Petersburg, The Herzen State Pedagogical University of Russia Publ., 2013. 190 p. [Computer music. V. 1. Creative work in computer music sphere].

41. Gorbunova I. B., Zalivadny M. S. Information technologies in music. Vol. 4. Music, mathematics, computer science. Textbook [Informatsionnye tekhnologii v muzyke. T. 4: Muzyka, matematika, informatika: Uchebnoe posobie]. St. Petersburg, The Herzen State Pedagogical University of Russia Publ., 2013. 190 p.

42. Gorbunova I.B. Information technologies in music. Vol. 3. Music computer. Textbook [Informatsionnye tekhnologii v muzyke. T. 3: Muzykal'nyi kompiuter: Uchebnoe posobie]. St. Petersburg, The Herzen State Pedagogical University of Russia Publ., 2011. 411 p.

43. Gorbunova I.B., Pankova A.A. [Komp'juternoe muzykal'noe tvorchestvo: teorija i praktika] – LAP LAMBERT Academic Publishing. Saarbrucken, Germany. - 2014. – 125 p. Kompiuternoe muzykal'noe tvorchestvo: teoria i praktika [Creative work in computer music sphere: Theory and practice].

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**Édesapja Paul Euler (1670–1745) kálvinista lelkész, anyja Margaretha Brucker (1677–1761), aki előkelő ősöket felvonultató családból származott. Leonhard Euler a svájci Bázelban született a házaspár első gyermekeként. Három testvére volt: két húga, Anna Maria (1708–1778), és Maria Magdalena (1711–1799), öccse Johann Heinrich (1719–1750) (ő később Georg Gsell (1673–1740) tanítványa lett, aki Szentpéterváron művészetet oktatott). Apja Leonhardot is lelkészi pályára szánta. Paul Euler barátja volt Johann Bernoulli matematikus, aki később Leonhard matematikai tanulmányaival is foglalkozott. Paul Euler diákkorában Jakob Bernoulli tanítványa volt. Apja kezdte el Leonhardot matematikai ismeretekre oktatni.

Bár Bázelben született, de egyéves korától a gyerekkora jelentős részét a szomszédos Riehenben töltötte, mivel apja ott prédikált, ezért szülei odaköltöztek. Itt járt iskolába, de ott csak a latin nyelvet oktatták, ezért apja „magántanárt” fogadott mellé, egy fiatal teológust, Johannes Burckhardt-ot (1691–1743), aki rajongott a matematikáért. Leonhard 1720-tól, tizennégy éves korától a bázeli egyetemen teológiát tanult. De ennél sokkal jobban érdekelte a matematika, amit a bázeli egyetemen nem tanítottak. Magánúton, matematikai könyvekből tanult. Már jó úton haladt, hogy apja kívánságának megfelelően lelkész legyen, amikor Johann Bernoulli észrevette Leonhard Euler matematikai tehetségét.

Lehetőségem nyílt rá, hogy bemutassanak a híres professzornak, Johann Bernoullinak. Nagyon elfoglalt volt, és kereken visszautasította, hogy magánórákat adjon nekem. De sokkal értékesebb volt az a tanács, hogy olvassak nehéz matematikai problémákkal foglalkozó könyveket abban a tempóban, ahogy nekem megfelel. Megengedte azt is, hogy ha akadályba vagy nehézségbe ütközöm, felkereshessem otthonában. Így gyakorlatilag minden vasárnap délután elmentem hozzá, és ő mindent elmagyarázott, amit magamtól nem értettem meg.

Johann Bernoulli meggyőzte Pault, hogy fia neves matematikus lehet. Az édesapja beleegyezett, hogy fia inkább matematikus legyen, így az eredetileg elkezdett tanulmányait 1723-ban (a másoknál szokásos tanulási időtartamnál másfél évvel hamarabb) befejezve megkapta a *primam lauream* fokozatot (ez nagyjából a mai érettséginek felel meg) és a *magister* címet, majd matematikát kezdett tanulni és abból 1726-ban kapta meg a végzést igazoló okiratot. 1727-ben a Francia Tudományos Akadémia által kiírt pályázaton, aminek témája „az árbócok legjobb elrendezése a hajókon”, második díjat nyert.

Daniel Bernoulli hívta 1727-ben a két évvel korábban létrehozott Szentpétervári Tudományos Akadémiára. 1731-ben a fizika professzora, majd két évvel később a matematikai osztály vezetője lett. Ez utóbbit Daniel Bernoullitól vette át, aki betegsége miatt visszaköltözött Svájcba. Ezekben az években Christian Goldbachhal is találkozott. 1734. január 7-én feleségül vette Katharina Gsellt, 13 gyermekük született, de mindössze 5 élte meg a felnőttkort.

1735-ben kezdődtek egészségi problémái. Ebben az évben egy súlyos láz majdnem a halálát okozta. 1740-ben a jobb szemére megvakult, de egy sikeres műtét visszahozta a látását. Később azonban újra elvesztette, és a műtét következtében 1771-ben a másik szemére is megvakult. (Megjegyzendő, hogy munkáinak kb. felét vakon hozta létre, amiben szentpétervári munkatársai önzetlenül segítették.)

1736-ban jelent meg Mechanica (Mechanika) című munkája, ami mérföldkő volt a tudományág életében, ugyanis Euler ebben mutatta be azokat a matematikai eszközöket, amiket a mechanika tanulmányozása során alkalmazni lehet. 1738-ban, majd 1740-ben is elnyerte a Francia Tudományos Akadémia nagydíját. Ekkor már elismert tudósnak számított.

1741-ben Nagy Frigyes porosz király hívására Berlinbe költözött, ahol részt vett a Berlini Tudományos Akadémia megszervezésében. Az Akadémia alelnöke és a matematikai osztály vezetője volt 1766-ig. Ekkor elhagyta Berlint, mivel az időközben az akadémiára érkező D'Alembert-rel képtelen volt együtt dolgozni.

Ezután ismét Szentpétervåron alkotott egészen 1783. szeptember 18-ig, amikor agyvérzés következtében meghalt.

**Munkássága**

Rendkívül termékeny és sokoldalú tudós, elsősorban matematikus, de kiváló fizikus is volt. Huszonnyolc nagyobb művet és több mint nyolcszáz értekezést írt. A matematika szinte valamennyi ágában maradandót alkotott.

- A számelméletben megtalálta a 8. tökéletes számot és 59 barátságos számpárt.[1]

- Bizonyította, hogy minden páros tökéletes szám alakú.
- Megmutatta, hogy az ötödik Fermat-szám összetett: $F_5$ osztható 641-gyel.
- Első publikált bizonyítását adta Fermat állításának: minden $4k$+1 alakú prímszám két négyzetszám összege.

- Ő jelölte először π-vel a kör kerületének és átmérőjének arányát, *e*-vel az sorozat határértékét, amit később róla neveztek el.

- Levezette az egyenlőséget.
- Az analitikus geometria keretében szinte egymaga megalkotta a ma használatos trigonometriát.
- 1748-ban megjelent könyvében szereplő koordináta-rendszernek két tengelye volt, melyeken már negatív értékek is szerepeltek. Gyakran használt polárkoordinátákat is.
- Síkgeometriában felfedezte és a nevét viseli a háromszög Euler-egyenese (1744).
- Felfedezte a Feuerbach-kört.

- Bizonyította a róla elnevezett Euler-tételt, mely összefüggést ad egy poliéder csúcsainak, éleinek és lapjainak száma között (1744).
- Elsőként haladta meg a kúpszeletek tárgyalása során Apollóniosz eredményeit.
- A gráfelmélet nyitányát jelenti a königsbergi hidak általa megoldott problémája.
- Megoldotta a karcsú rudak rugalmas kihajlásának problémáját.
- A hidrodinamikát ma is az ő felfogásában tárgyalják.
- Az örvényszivattyúk és turbinák méretezését ma is az Euler-turbinaegyenlet szerint végzik.
- A pörgettyűmozgást az Euler-féle kinetikai egyenletek segítségével vizsgálta.
- Foglalkozott valószínűségszámítással, komplex számokkal, harmonikus sorokkal, moduláris aritmetikával, differenciálegyenletekkel.
- A csillagászatban foglalkozott a bolygók pályáinak kiszámításával.
- Az optikában a kromatikus aberráció matematikai elemzésével.
- Írt könyvet a hidraulikáról, hajótervezésről, tüzérségről, zenéről. Jelentős térképészeti munkát is végzett.

Foglalkozott a tudományos módszerek népszerűsítésével is. 1768 és 1772 között jelent meg háromkötetes, *Levelek a német hercegnőhöz* címet viselő munkája, ami ezeket a népszerű tudományos témákkal foglalkozó írásait tartalmazza.

Halálakor 560 megjelent műve volt, posztumusz cikkeit a Szentpétervári Akadémia folyamatosan adta ki. 1843-ban, amikor úgy tűnt, mindet feldolgozták, a lista 756 tagot tartalmazott. Ekkor váratlanul 61 kéziratot találtak. A huszadik század elején összeállított listán 866 írás van.

(Forrás: wikipédia)